ILL-POSEDNESS ISSUES FOR SOME NONLINEAR DISPERSIVE EQUATIONS

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In general, the Cauchy problem for nonlinear dispersive wave equations is proven to be locally well-posed by a Picard type iteration scheme in a suitable function space, on the integral (Duhamel) formulation. This function space can be a Strichartz space (nonlinear Schrodinger equations), a Bourgain space (Korteweg-de Vries or Kadomtsev-Petviashvili II equation). This process leads immediately to the smoothness of the flow map.

The aim of this talk (based on joint papers with Luc Molinet and Nikolay Tzvetkov), is to prove the rather surprising fact that this procedure fails for some familiar nonlinear dispersive equations such as the Kadomtsev-Petviashvili I equation, the Benjamin-Ono equation and a class of non-local equations, for any reasonable spaces of initial data (Sobolev classes of any order, energy space,...). A related fact is that the flow maps associated to the solutions obtained by compactness methods will never be smooth.

The construction of counter-examples lies on the non triviality of some “resonant manifolds” associated to the symbol of the linear part.