Critical and subcritical fractional Trudinger-Moser type inequalities on $\mathbb{R}$

Futoshi Takahashi (futoshi@sci.osaka-cu.ac.jp)
Osaka City University

Abstract. In this talk, we concern with the critical and subcritical Trudinger-Moser type inequalities for functions in a fractional Sobolev space $H^{1/2,2}$ on the whole real line. More precisely, put

$$A(\alpha) = \sup_{\substack{u \in H^{1/2,2}(\mathbb{R}) \setminus \{0\} \\ \|(-\Delta)^{1/4} u\|_{L^2(\mathbb{R})} \le 1}} \frac{1}{\|u\|^2_{L^2(\mathbb{R})}} \int_{\mathbb{R}} \left(e^{\alpha u^2} - 1 \right) dx,$$

$$B(\alpha) = \sup_{\substack{u \in H^{1/2,2}(\mathbb{R}) \\ \|u\|_{H^{1/2,2}(\mathbb{R})} \le 1}} \int_{\mathbb{R}} \left(e^{\alpha u^2} - 1 \right) dx,$$

where $\|u\|_{H^{1/2,2}(\mathbb{R})} = \left(\|(-\Delta)^{1/4} u\|^2_{L^2(\mathbb{R})} + \|u\|^2_{L^2(\mathbb{R})} \right)^{1/2}$ denotes the full Sobolev norm of $H^{1/2,2}(\mathbb{R})$. Then we prove

$$A(\alpha) = \begin{cases} < \infty, & \alpha < \pi, \\ = \infty, & \alpha \ge \pi, \end{cases}$$

and

$$B(\alpha) = \begin{cases} < \infty, & \alpha \le \pi, \\ = \infty, & \alpha > \pi. \end{cases}$$

Also we show that a relation of two supremums,

$$B(\pi) = \sup_{\alpha \in (0,\pi)} \frac{1 - (\alpha/\pi)}{(\alpha/\pi)} A(\alpha).$$

For the attainability of two supremums, we prove that $A(\alpha)$ is attained for any $\alpha \in (0, \pi)$, while $B(\alpha)$ is not attained for small $\alpha > 0$.

This talk is based on a paper [1]

References

[1] F. Takahashi: *Critical and subcritical fractional Trudinger-Moser type inequalities on* $\mathbb{R}$, to appear in Advances Nonlinear Anal. (arXiv:1702.08206)